Classical Test Theory
Main concepts

Christophe Lalanne
ch.lalanne@gmail.com

November, 2009

Summary

❝ abstract here. . . ❞

Outline

1.

2.

3.

4.

Various functions used throughout this chapter were collated in the package **Psychomisc**.

[1]

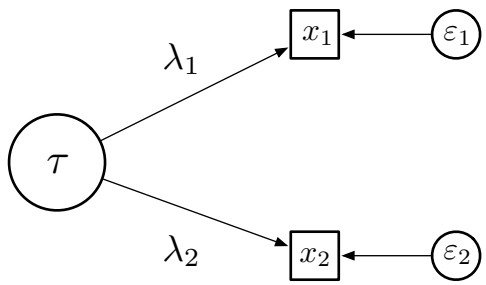
The True score model

Types of test

Several tests are said to be *congenerically* equivalent if all tests may be expressed in terms of one factor and a residual error.

Parallel tests are the special case where (usually two) tests have equal factor loadings ($\lambda_1 = \lambda_2$ in the diagram below).

Tau equivalent tests have equal factor loadings but may have unequal errors. Congeneric tests may differ in both factor loading and error variances.



Formalisation

Let's write the assumptions used to define these models:

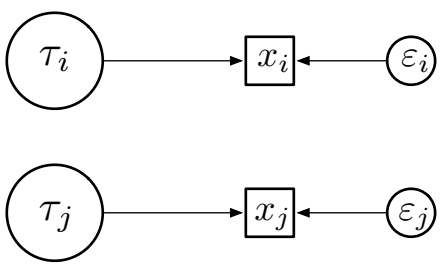
(a_1) τ-equivalence, $\tau_i = \tau_j$

(a_2) essential τ-equivalence, $\tau_i = \tau_j + \lambda_{ij}, \quad \lambda_{ij} \in \mathbb{R}$

(a_3) τ-congenerity, $\tau_i = \lambda_{ij0} + \lambda_{ij1}\tau_j, \quad \lambda_{ij0}, \lambda_{ij1} \in \mathbb{R}, \lambda_{ij1} > 0$

(b) uncorrelated errors, $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0, \forall i \neq j$

(c) equal error variances, $\mathbb{V}(\varepsilon_i) = \mathbb{V}(\varepsilon_j)$



Formalisation (Con't)

Now, we see that:

- Parallel tests are defined by Assumptions (a_1), (b) and (c),
- Essentially τ-equivalent tests are defined by Assumptions (a_2) and (b),
- Congeneric tests are defined by Assumptions (a_3) and (b).

Congeneric Test

```
require(psych)
test <- sim.congeneric(short=FALSE, N=100)
round(cor(test$observed),2)
```

Raw correlation between sum scores and latent scores is 0.85, and R^2 of scores with factors is 0.87.

The first principal axis accounts for 45% of the variance in sum scores.

	I1	I2	I3	I4	PA	h^2
I1	—				0.91	0.83
I2	0.64	—			0.71	0.50
I3	0.48	0.39	—		0.53	0.28
I4	0.42	0.30	0.22	—	0.44	0.20

circumplex structure

In summary

Pros:

- provides useful guidelines for item writing/checking/revision and facilitates field-testing of the instrument using small samples
- few but simple mathematical formulation, with straightforward estimation of model parameters; this yields scoring rules of practical interest
- a class of 'weak' model because its assumptions are easily met

In summary

Cons:

- At the item level, CTT is mostly organized around the difficulty and discrimination parameters which are sample dependent: higher item difficulty would be obtained from an examinee samples of lower-average knowledge, and higher item discrimination would occur from an heterogeneous examinee sample.

- At the test level, difficulty directly affects the test scores, and the true score model does not allow specific pattern response (sum score is a sufficient statistic): As a consequence, it is not possible to predict one's performance on another test.

References

[1] P M Fayers and D Machin. *Quality of life. The assessment, analysis and interpretation of patient-reported outcomes*. Wiley, 2000.

– Typeset with FoilTEX (version 2)